Journal of Quantitative Linguistics
2006, Volume 13, Numbers 2–3, pp. 177–193
DOI: 10.1080/09296170600850106

Routledge
Taylor & Francis Group

Fractal Geometry of Texts: An Initial Application to the Works of Shakespeare*

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ABSTRACT

It has been demonstrated that there is a geometrical order in text structures. Fractal geometry, as a modern mathematical approach and a new geometrical standpoint on natural objects including both processes and structures, is here employed for textual analysis. For this first study, the works of William Shakespeare were chosen as the most important items in Western literature. By counting the number of letters in a text, it is possible to study the whole text statistically. A novel method based on basic assumption of fractal geometry is proposed for the calculation of fractal dimensions of texts. The results are compared with Zipf's law. Zipf's law is successfully used for letters instead of words. Two new concepts – namely Zipf's dimension and Zipf's order – are also introduced. It can be seen that changes of both fractal dimension and Zipf's dimension are similar, and dependent on the text length. Interestingly, directly plotting the data obtained in semi-logarithmic and logarithmic forms also leads to a power-law.

INTRODUCTION

Since the revolutionary discovery of fractal geometry by Mandelbrot (1983) numerous studies in different branches of science have been undertaken to understand fractality of different objects and processes. There are several differences between fractal geometry and Euclidean geometry. Firstly, the recognition of fractals is very recent (from such new classification point of view), as they have only been formally studied in the past two decades, whereas Euclidean geometry goes back over

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2000 years. Secondly, whereas Euclidean shapes normally have a few characteristic sizes or length scales (e.g., the radius of a circle or the length of the side of a cube), fractals have various characteristic sizes. Fractal shapes are self-similar and independent of size or scale. Thirdly, Euclidean geometry merely provides a good description of manmade objects; whereas fractals can be used as a representation of naturally occurring geometries. It is likely that this limitation of our traditional language of shape is responsible for the persistent difference between man-produced objects and natural shapes. Finally, Euclidean geometries are defined by algebraic formulae. Fractals are normally the result of an iterative or recursive construction or algorithm.

It has been reported that most objects in nature and manmade items are fractal on both microscopic and macroscopic scales (Barnsley, 1993; Peitgen & Saupe, 1988; Moon, 1992; Falconer, 1990). Indeed, an important and interesting feature of fractal geometry is related to its utility for analysing natural structures and even different processes. Therefore, fractal analyses have been used in various branches of science. The most vivid work using fractals can be found in chemistry and physics (Rothschild, 1998; Avnir, 1989; Avnir et al., 1984; Avnir et al., 1992; Pfeifer, 1986; Kopelman, 1988; Eftekhari, 2003a), as it is related to three different typical features: fractals have been found at chemically reactive surfaces, in molecular structures, and even during a chemical reaction. The first and second of these features relate to the usual usage of the term "fractal", as they concern the fractal structure of 'surfaces'. The third feature is related to fractal "processes", suggesting that chemical reactions may have fractal dimensions.

Fractals are of interest in art as well as science; this is due to both artistic and scientific investigations of art. Fractal shape creations can be incorporated into art and vice versa. The appearance of fractality processes is also linked to a long artistic history. This emerges from the First Art, music, which, although a man-made artefact, can be extended to natural phenomena. Since the discovery of fractal geometry, the question of whether music has fractal geometry has been a matter of debate (Voss & Clarke, 1975; Campbell, 1986; Schroeder, 1987; Campbell, 1987; Voss & Clarke, 1978). Hsu and Hsu (1990) report a method for calculating the fractal dimension of music, and answer this question in the affirmative. They recognize an inverse log–log relationship between the frequency and intensity of natural events

(Hsu & Hsu, 1990, 1991, 1993). Based on an analysed melody in terms of the interval between successive pitches, they use the following formula:

$$F = c/i^D \tag{1}$$

where D is the fractal dimension of the composition, i the interval between two successive pitches, F the percentage frequency of i and c is a constant proportionality factor. This method has been successfully used for analysing and calculating the fractal dimension of different types of music. This is an interesting area of research in both scientific and artistic terms. Several types of music have been analysed to find their fractal structures (Hsu & Hsu, 1990, 1991, 1993; Bigerelle & Iost, 2000), and several music items have been created fractally (Bolognesi, 1983; Dodge & Profile, 1988; West & Shlesinger, 1978; Thomsen, 1980) using the basic assumptions of fractal geometry.

Similarly, considerable attention has been paid to the fractal analysis of language (Hrebicek, 1995; Köhler, 1997), since texts can be well-analysed by statistical methods. In the context of the present paper, we would like to introduce fractal structure to another object that is of interest both scientifically and artistically. This can be considered a well-defined approach, as it is based on the simple assumptions of fractal geometry as they have been elaborated for music. Observing the similarity of music and literature, both artistic and structural, we apply the above-mentioned method to the analysis of texts. This is simply achieved by treating letters in texts as equivalent to notes in music. Thus, we can use equation (1) to calculate the fractal dimension of texts. In order to preserve the original form of the equation, we will use the same symbols but with different meanings. Therefore, D is the fractal dimension of texts, i the interval between two letters in alphabetical series, F the percentage of i and c is still a proportionality factor.

A problem which must be taken into account is the description of i for texts. For music, i is the interval between notes; a typical note is chosen as the base note, and the value of i for other notes is calculated relative to this base note. Thus, the base note has $i = 0$. We modify this approach for alphabetical letters by introducing a theoretical letter before A with incidence of zero. By choosing this non-existent letter as the base letter, we can calculate the values of i for other letters (indeed, all letters). For the base letter, which can have no role in data analysis, ($i = 0$); for A ($i = 1$), ..., and for Z ($i = 26$). The advantage of this modification is that the value of i for each letter is equal to that letter's rank in the alphabetical series. It should be emphasized that it is necessary to obey rules from literature on using alphabetical order for the letters utilized in texts. This allows us to compare the results obtained from the fractal analysis of texts with those obtained from other statistical methods.

It is worth noting that using alphabetical order in texts has no physical meaning, in a similar way to musical notes. Alphabetical order is, indeed, an artificial order imposed by creators of language (i.e., human beings). However, using this artificial standard is useful in order to understand the texts in accordance with the alphabet of the language of the text. On the other hand, although this method is applicable for languages with well-defined alphabetical order, it cannot be easily utilized for some complex languages such as Chinese, Old Egyptian, Accadian, Maya, etc. However, this procedure leads to a better understanding of alphabetical order. As will be described later, this can be used to understand statistical orders such as Zipf's order.

RESULTS

Fractal Analysis

Hamlet, as a famous Shakespeare tragedy, was selected as a typical example text. As expected, the appearance of different letters in the text has a chaotic arrangement. This is observable in Figure 1, where the number of appearances of each letter is given for the text of *Hamlet*. The characteristic data for this text are presented in Table 1. According to equation (1), the fractal dimension of the text can be determined from slope of the F versus $1/i$ curve plotted in a log – log scale. The corresponding plot is illustrated in Figure 2. Although the data are dispersed, the slope of the curve can be determined using a mean square root approach. Consequently, the curve slope suggests a fractal dimension of 0.45 for *Hamlet*.

As can be seen, the fit of the curve is weak and has a low determination coefficient (R^2) of 0.06712; this is far from being a well-defined fit to the data. This reflects the extensive dispersion of data around the curve. It goes back to the limitation of letters in text structure. Of course, this is usual for this type of fractal structure. In fractal music, the data fit obtained from the notes is also weak, compared with the usual curves

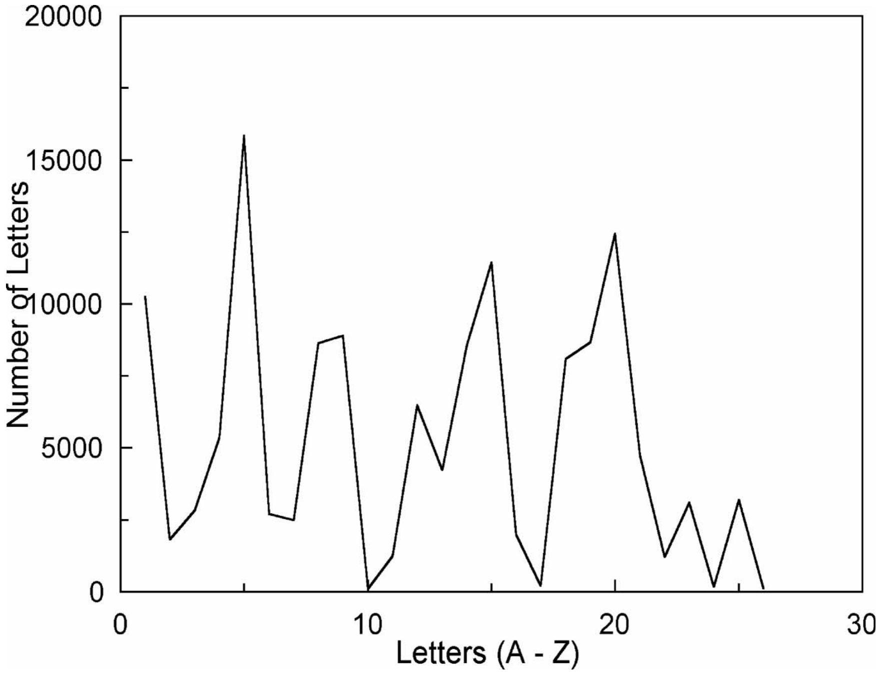

Fig. 1. Appearance of letters in a typical Shakespeare work, *Hamlet*.

found in scientific works. This is indicative of the degree of fractality (Eftekhari, 2004).

Degree of Fractality

Two different features have been elaborated from the concepts of fractal geometry (Mandelbrot, 1983):

1. The objects are more complicated than those defined by Euclidean geometry (with simple integer dimensions), and they have non-integer dimensions depending on their complexity.
2. The complex objects can be defined in terms of self-similarity or self-affinity. We are aware of various fractal models and real objects which have simple patterns of self-similarity or self-affinity.

However, most real fractal objects do not follow a well-defined fractal pattern. Indeed, the revolutionary feature of fractal geometry is the first of the features mentioned above, which made it a universal theory. It is

Table 1. Characteristics data for letters applied in *Hamlet*.

Letter	Letter interval (i)	Incidence	Incidence % (F)
*Base letter**	0	0	0
A	1	10,251	7.5931645963423
B	2	1,816	1.34515529284534
C	3	2,840	2.10365695577135
D	4	5,375	3.98139300608135
E	5	15,845	11.7367762197877
F	6	2,712	2.0088442479056
G	7	2,493	1.84662563054154
H	8	8,639	6.399117056658
I	9	8,905	6.59614971519152
J	10	110	0.0814796708221299
K	11	1,257	0.931090420212884
L	12	6,489	4.80655985422546
M	13	4,239	3.1399302237728
N	14	8,578	6.35393287556573
O	15	11,450	8.48129300830352
P	16	2,006	1.48589290608357
Q	17	218	0.161477893083857
R	18	8,100	5.99986666962956
S	19	8,668	6.42059806078384
T	20	12,450	9.2220172885047
U	21	4,738	3.50955163959319
V	22	1,219	0.902942897565239
W	23	3,110	2.30365251142567
X	24	177	0.131108197595609
Y	25	3,198	2.36883624808338
Z	26	120	0.0888869136241417

*This is a theoretical character introduced to calculate the value of i for 26 letters of the Latin alphabet. All intervals were estimated in accordance with this theoretical letter, but it has no role in the statistical analysis of the text.

obvious that all objects are subject to this first feature of fractal geometry. In fact, it is very hard to find an object with an integer dimension. For example, it is difficult to find a completely smooth surface without any roughness (even at the microscopic scale), describing an integer dimension of 2.

The question may be raised: what is the difference between fractal and non-fractal surfaces? The question can be answered simply by introducing the fractality factor, *viz*. zeta-function, which is a dimensionless factor between 0 and 1 (and may also be presented in percentage) to calculate how much the surface is defined by the fractal patterns (Eftekhari, 2004). To understand the physical meaning of zeta-function, we describe it for a classic fractal object with a known fractal pattern, the Sierpinski gasket (Fig. 3). The Sierpinski gasket is one of the most famous fractal patterns with a known fractal dimension, $\log(3)/\log(2) = 1.58496$ (Mandelbrot, 1983).

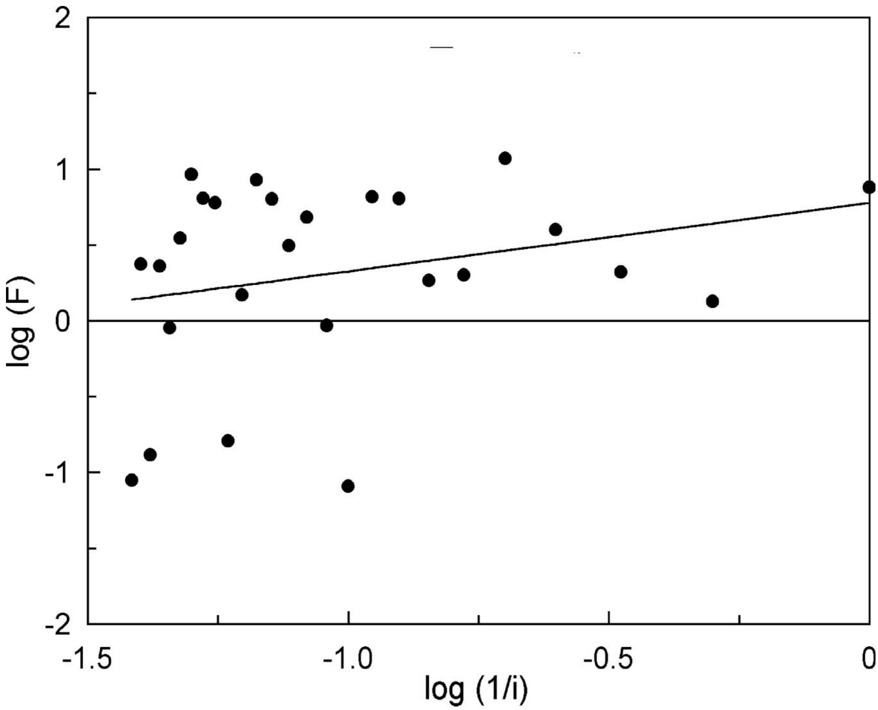

Fig. 2. Plotting F against $1/i$ in a log–log scale for a typical Shakespeare work, *Hamlet*.

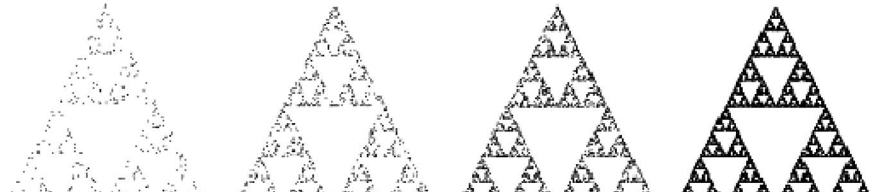

Fig. 3. Degree of fractality presented for a series of Sierpinski gaskets.

According to the Sierpinski gasket pattern, all four gaskets displayed in Figure 5 have the same fractal dimension of 1.58496. But do they have the same fractality? If an artificial object fabricated on the basis of the Sierpinski gasket has ideal pattern of the right gasket, the object can be exhausted and changed to the left gasket as the result of the progress of time or any other destrictive factor, such as corrosion. Although the fractal dimensions of all the gaskets are the same, they lose their fractality from right to left, which can be expressed by a decreasing zeta-function.

Therefore, the value of the zeta-function indicates the degree of fractality for an object. For an ideal fractal pattern, the value of zeta-function approaches unity (100% fractality), but this value is lesser for real objects. When we design a fractal object like a Sierpinski gasket theoretically on paper, the value of zeta-function is 1, suggesting 100% fractality of our mathematical pattern. However, if we fabricate this pattern on a gold surface (i.e., the most reliable method for the fabrication of artificial fractal electrodes) we can never have 100% fractality. Although a gold surface is very smooth, the roughness factor (the ratio of real surface area to geometrical area) even for a very smooth gold surface is about 1.2. Consequently, the roughness of the gold surface changes the desired fractality. As shown in an investigation of a series of real and artificial gold electrodes by electrochemical methods (Eftekhari, 2004), the determination coefficient of a data fit can be considered as a factor of the surface fractality, or the value of the zeta-function.

To clarify this point, it should be emphasized that the fractal dimension of real objects is not completely scale-independent, as predicted by fractal geometry. In other words, a fractal structure varies as the scale changes. For instance, by zooming in using a microscope, we can see severe changes in a fractal structure. In a solid investigation, it has been demonstrated that the fractal structures of surfaces are significantly different at micro- and nano-scale (Eftekhari et al., 2005).

This is also true for the system under investigation. Indeed, the low determination coefficient reported indicates weak fractality in the text of *Hamlet*. This is not strange, as we cannot expect to find strong fractality in literature. If we take into account the case of music, which is very similar to the case under consideration, we can say that fractally generated music with a computerized approach has strong fractality with a zeta-function approaching 1; whereas common kinds of music also have fractal structures, but with weak fractality. Indeed, we can consider fractally created music to have a well-defined fractal pattern like a

Sierpinski gasket as presented on the right side of Figure 3, and conventional music can be considered to have a fractal pattern like the gasket on the left side. Similarly, we can consider a text (e.g., a Shakespeare play) as a fractal object, like the Sierpinski gasket presented on the left in Figure 3. However, in contrast to the case of fractally created music, we have no fractally created manuscript to be considered that will have a well-defined fractal pattern (such as the right-hand-side Sierpinski gasket). Discussing this problem of fractality demonstrates that weak data fitting for a fractal analysis of texts is to be expected, and that the fractal dimension calculated for *Hamlet* is of interest, as well as those of other fractal objects.

It should also be emphasized that the value estimated for the fractal dimension of *Hamlet* is not the purpose of this study. This is the first paper in this area and aims to show the power of the method proposed here for the mathematical study of literature. The method is reliable, as it is based on the simple assumption of fractal geometry and, as mentioned above, the hypothesis is compared with music, which is a well-known example of fractals. The results obtained from an analysis of different Shakespeare tragedies are summarized in Table 2.

Although the values estimated for the fractal dimensions of the different texts are in the same range there are, however, significant differences between them. It is known that the value of the fractal dimension can be used for comparative studies of similar objects. For example, if we prepare some gold surfaces by the deposition of gold under different conditions, the fractal dimensions of such different

Table 2. Calculated values for the fractal dimensions of different Shakespeare tragedies.

Tragedies	Number of total letters	Fractal dimension (D_f)	Determination coefficient (R^2)
Anthony and Cleopatra	116,209	0.5516	0.07923
Coriolanus	124,626	0.4707	0.06009
Hamlet	135,003	0.4500	0.06712
Julius Caesar	86,659	0.5269	0.07023
King Lear	115,986	0.5598	0.07934
Macbeth	77,524	0.5985	0.09261
Othello	115,245	0.5699	0.08031
Romeo and Juliet	105,834	0.5496	0.08686
Timon of Athenes	83,500	0.5358	0.07680
Titus Andronicus	92,467	0.5180	0.06743

Au-deposits are an excellent factor for comparison of the surface structures (Eftekhari, 2003b, 2003c).

Zipf's Law

As stated above, the results are not satisfactory in comparison with other fractals we know. It is useful to compare this fractal analysis of Shakespeare's works with statistical analyses based on other approaches. Methods in this context are usually based on analysis of words rather than letters. The most famous method for this purpose is based on Zipf's law, which was proposed by G. K. Zipf (1965). Zipf's law suggests that frequency of occurrence of some event (P), as a function of the rank (i) when the rank is determined by the above frequency of occurrence, is a power-law function $P_i \sim 1/i^a$ with the exponent a close to unity. The similarity of this relation to that noted in Equation (1) is obvious. Zipf's law has been widely used for statistical analysis of texts (Guiter & Arapov, 1982; Rousseau & Zhang, 1992; Li, 1992; Perline, 1996; Troll & Graben, 1998; Prün, 1999; Cancho & Sole, 2002; Montemurro & Zanette, 2002; Roelcke, 2002). The most famous example in this context is the statistical analysis of the most frequently occurring words such as "the", "of", "to", etc. Let us now examine Zipf's law based on the strategy of this study, i.e. analysing the letters of a whole text. The results for *Hamlet* are shown in Figure 4. The data are similar to those presented in Figure 1; however, as the letters are ordered in accordance with their frequency of occurrence, the changes are monotonic. The letters indicated on the curve show the arrangement of letters in accordance with their frequencies of occurrence, following Zipf's law. We call this arrangement "Zipf's order". If we transform this curve (Fig. 4) into a log–log scale (similar to that in Fig. 2), we can estimate the value of a in Zipf's law, which is called Zipf's dimension D_Z. The results obtained for different Shakespeare tragedies are summarized in Table 3. The required Zipf's orders for different tragedies are also illustrated in Figure 5.

As seen, the Zipf's orders for different tragedies are very similar, and it suggests that Zipf's law can be used as a general method for statistical analysis of texts based on letter counting, as the frequency of each letter is approximately constant in English language. Indeed, this suggests that increase of the letter frequencies obey a power-law. On the other hand, the data presented in Table 3 suggests an ideal behaviour of the result accompanied by good determination coefficients (when the letters are

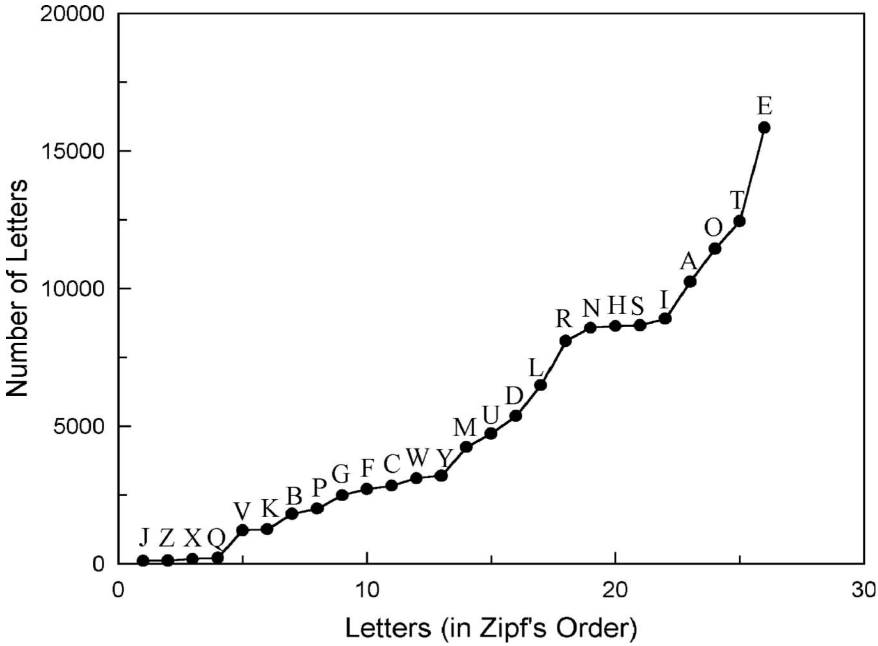

Fig. 4. The numbers of each letter repeated in *Hamlet* as a function of letters ordered due to their intervals in Zipf's order.

Table 3. Analysis of different Shakespeare tragedies based on Zipf's law.

Tragedies	Zipf slope*	Determination coefficient (R^2)	Zipf dimension[†] (D_Z)	Determination coefficient (R^2)
Anthony and Cleopatra	0.40587	0.921137	1.9320	0.972011
Coriolanus	0.40876	0.927299	1.8764	0.955026
Hamlet	0.40585	0.928312	1.6973	0.954612
Julius Caesar	0.40596	0.948025	1.9448	0.956628
King Lear	0.40276	0.913496	1.9424	0.955115
Macbeth	0.40247	0.928162	1.9414	0.974379
Othello	0.41361	0.925255	1.9720	0.961605
Romeo and Juliet	0.40588	0.920359	1.8441	0.977931
Timon of Athenes	0.41199	0.927070	1.8935	0.959367
Titus Andronicus	0.40869	0.936204	1.9558	0.961202

*Slope of the F (percent of incidence) – Letters (arranged in Zipf's order) plot.
[†]Estimated from the slope of the log (F) — log ($1/n$) plot, where n refers to the letter in Zipf's order. Indeed, D_z is the exponent of a in the Zipf's equation.

Anthony and Cleopatra:	Z J Q X K V B G F P W Y C M U D L H N I R S O T A E
Coriolanus:	Q J X Z K V P G B F Y W C M D L U H R N S A I O T E
Hamlet:	J Z X Q V K B P G F C W Y M U D L R N H S I A O T E
Julius Caesar:	Q J X Z K V P G B F W Y C M D L U H N R I S O A T E
King Lear:	Z Q J X V K B P F C G W Y M U D L I H S R N A O T E
Macbeth:	Z J X Q V K P G B Y F W C M U L D R I S N H A O T E
Othello:	Z Q J X K V P B G F C W Y M U D L R N H S I A T O E
Romeo and Juliet:	Z Q X J K V P B G F C W Y M U D L N R S H I A O T E
Timon of Athenes:	Z Q J X K V B G P C F Y W M U D L R H N I S A O T E
Titus Andranicus:	Z X J Q K V P G B F C Y W M L D U H I R N S A O T E

Fig. 5. Zipf's order of letters for different Shakespeare tragedies.

ordered with Zipf's order). Although it is clear that Zipf's dimension is different from the fractal dimension estimated previously (reported in Table 2), it can be concluded by comparison of the data in Tables 2 and 3 that both dimensions proposed for literature are commensurate. This means that the fractal dimension of a text (as investigated for Shakespeare's tragedies) is higher than others when its Zipf's dimension is higher than others, and *vice versa*. As illustrated in Figure 6, the changes of both fractal dimension and Zipf's dimension are the same. In other words, both fractal dimension and Zipf's dimension can be used equivalently for comparative investigation of different texts.

Interestingly, it is obvious that the dimension (both Zipf and fractal) of the texts is dependent on the text length. For the 10 tragedies of Shakespeare under investigation, the dimension is higher for the shorter texts. The dependence of Zipf's law on text length has also been previously reported (Debowski, 2002). Although there is an exception at about 100,000 letters-text, this behaviour is also applicable for shorter texts. The equivalency of the fractal dimension and Zipf's dimension indicates that although literature has weak fractality it does, however, have a certain structure of fractality, as Zipf's law is a known approach for textual analysis.

Direct Data Plotting

Both above-mentioned approaches for text analysis are based on reciprocal power-laws. To use this type of power-law, $1/i$ or $1/n$ were utilized. Now, it is appropriate to make a direct statistical analysis of the literature. Once again, we use *Hamlet* as a typical example. We use different forms of semi-logarithmic and logarithmic data plotting. Figure 7 depicts the results obtained from P versus log (i), log (P)

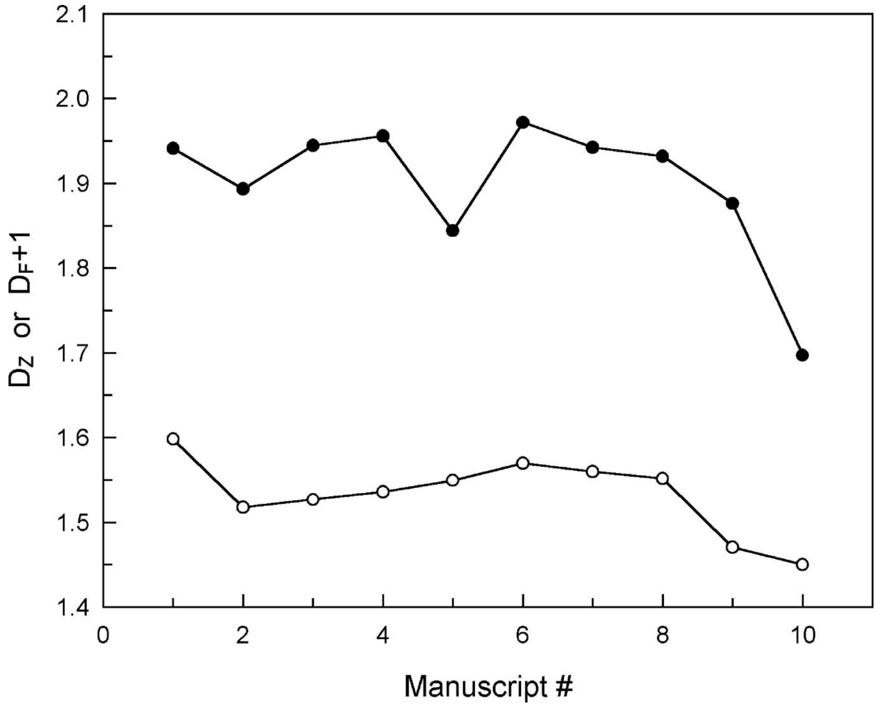

Fig. 6. Variations of both Zipf's dimension D_Z and fractal dimension D_F (fractal dimension was illustrated one unit higher as $D_F + 1$ for purposes of comparison) for different texts. Manuscript # refers to different Shakespeare plays investigated in this research arranged in accordance with their lengths (number of letters).

versus *i*, and log (*P*) versus log (*i*). To find the best method for data plotting, the determination coefficients for the methods are compared. The values obtained for determination coefficients of the three methods were 0.675249, 0.846831 and 0.954612, respectively. Similar results were also obtained for the other Shakespeare works.

Interestingly, the data plotting based on full logarithmic (log – log) form provided the best result. This suggests that a power-law function is an appropriate approach to text analysis and even produces better results, compared to other statistical methods. This power-law function is similar to Zipf's law, as using *n* or $1/n$ leads to similar results. In fact, the data plotting illustrated in Figure 7(c) can be considered as a kind of Zipf's law. Indeed, the results obtained are indicative of the fact that

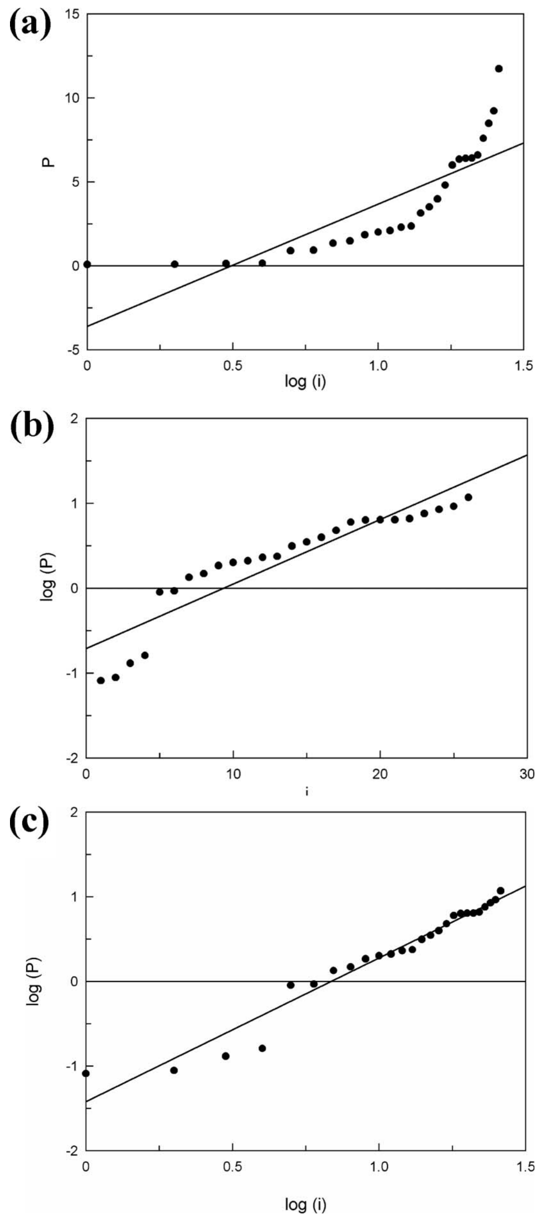

Fig. 7. (a). Plotting the data obtained from *Hamlet* in different forms: (a) $P - \log(i)$, (b) $\log(P) - i$, (c) $\log(P) - \log(i)$. The percent of incidence (P) was obtained from the data reported in Table 1 and the rank i is the Zipf's order reported in Figure 5.

Zipf's law based on alphabetical letters is an appropriate statistical method for analysis of texts.

CONCLUSION

It has been demonstrated that a statistical analysis of letters based on fractal geometry or Zipf's law can be used to investigate texts. Two novel features have been proposed in this study for the first time: (i) it has been shown that literature can have fractality and (ii) the usefulness and powerfulness of Zipf's law for the analysis of literature based on all the letters in a text has been demonstrated. In this regard, the degree of fractality, and new concepts such as Zipf's order and Zipf's dimension, has also been described. In conclusion, this paper proposes a new strategy for the analysis of written materials. Contrary to currently available methods, which are based on analysis of words and are just applicable for texts with meaning in a certain language, the method proposed here can be used as a general approach even to random texts. In addition, the relationship of fractality and Zipf's law has also been shown. Of course, both fractal geometry and Zipf's law yield similar results in the statistical analysis of texts, but their underlying concepts are different. Fractal analysis is in accordance with alphabetical order, but Zipf's law is in accordance with frequency of occurrence. This is the reason that the fractal dimension and Zipf's law are different; but both can be used as numerical measures for the comparative study of texts.

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